E-ISSN: 2349-7017
P-ISSN: 2349-7009
https://www.ijiris.com/archives
https://doi.org/10.26562/ijiris.2026.v1205.01

A Study of the Interval Newton Method for Nonlinear Equations

G.Upender Reddy*
Associate Professor, Department of Mathematics,
Mahatma Gandhi University, Nalgonda, India
upendermathsmgu@gmail.com
https://orcid.org/0000-0002-8460-0489
ROR https://ror.org/05n7bzj69
M.Varsha,P.Chamanthi,Ch.Shivaranjani,
M.Sc., Students. Dept of Mathematics,
Mahatma Gandhi University, Nalgonda, India
varshamanimadde@gmail.com, chamanthipeddagoni@gmail.com, shivaranjani681@gmail.com
ROR https://ror.org/05n7bzj69

Publication History
Manuscript Reference No: IJIRIS/RS/Vol.12/Issue05/ISJY26.ISJY10080
Research Article | Open Access | Double-Blind Peer-Reviewed| Article ID: IJIRAE/RS/Vol.12/Issue05/ISJY26.ISJY10080
Received: 14, April 2026, Revised: 02, May 2026, Accepted: 08, June 2026, Published Online: 03, July 2026.
https://www.ijiris.com/volumes/Vol12/iss-05/01.JYIS0080.pdf
Article Citation:Upender,Varsha,Chamanthi,Shivaranjani(2026),A Study of the Interval Newton Method for Nonlinear Equations. IJIRIS: International Journal of Innovative Research in Information Security, Volume 12, Issue 05 of 2026 pages 327-331 **Doi:>** https://doi.org/10.26562/ijiris.2026.v1205.01
BibTeX Key: Upender@2026Nonlinerar
IJIRIS papers should be cited as IJIRIS(International Journal of Innovative Research in Information Security, AM Publications, India 2026, ISSN 2349-7017, https://doi.org/10.26562/ijiris.2026.v1205.01 The journal's official abbreviation is IJIRIS. Orcid: https://orcid.org/0009-0004-9398-7488

Abstract: This paper presents a study of the Interval Newton Method, an important numerical technique for obtaining verified solutions of nonlinear equations. Unlike classical methods such as the Newton Raphson method, the Interval Newton Method provides guaranteed intervals containing the true roots. The paper introduces the fundamentals of interval arithmetic and derives the method using the Mean Value Theorem. Several numerical examples involving algebraic and transcendental equations are discussed to illustrate the efficiency and reliability of the method. The study shows that the Interval Newton Method produces accurate root enclosures with fast convergence and plays a significant role in verified numerical computation and scientific applications.
Keywords: Interval Newton Method, Interval Arithmetic, Interval Analysis, AMS Subject Classifications (2020): 65G40, 65H10**,** 65G20**,** 65H05.

1. INTRODUCTION

The solution of nonlinear equations is an important area in mathematics, science, and engineering. Many real-world problems in physics, economics, and engineering can be represented in the form f(x) =0. While simple equations may have exact analytical solutions, higher-degree polynomial and transcendental equations often require numerical methods for obtaining approximate solutions. Classical numerical methods such as the Bisection Method, Newton–Raphson Method, and Secant Method are commonly used because of their simplicity and efficiency [5]. However, these methods do not always guarantee that the computed approximation contains the true root, and some methods may fail to converge if the initial guess is poor. To address these limitations, interval-based methods were developed. Among them, the Interval Newton Method is a powerful technique that combines Newton iteration with interval arithmetic to provide guaranteed enclosures of roots. Instead of producing only an approximate value, the method generates an interval that is guaranteed to contain the true solution. The main objective of this paper is to study the Interval Newton Method, including its mathematical foundation, implementation, and applications. The method is applied to different nonlinear equations to demonstrate its accuracy, efficiency, and reliability.

1.1 Definition: A mathematical equation of the form f(x) = 0 which involves algebraic terms is called algebraic equation. A mathematical equation of the form f(x) = 0 which involves transcendental functions is called transcendental equation.

1.2. Newton–Raphson Method: The Newton–Raphson Method is a powerful numerical technique used to solve nonlinear equations by utilizing the derivative of the function. Starting from an initial approximation x_n, the method generates improved approximations using the iterative formula $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$. One of the main advantages of this method is its quadratic convergence, which means that the approximations approach the root very rapidly when the initial guess is close to the actual solution. Geometrically, the method can be interpreted as drawing a tangent line to the curve y=f(x) at the point $(x_n,f(x_n))$; the point where this tangent intersects the xxx-axis becomes the next approximation

E-ISSN: 2349-7017
P-ISSN: 2349-7009
https://www.ijiris.com/archives
https://doi.org/10.26562/ijiris.2026.v1205.01

x_{n+1}. Because of its speed and efficiency, the Newton–Raphson Method is widely used in scientific and engineering computations.

1.3 Definition: Interval

An interval [a, b] is the set of all real numbers between a and b where a<b. Let X = [a, b] and Y = [c, d] be two intervals.

(i). Addition of two intervals is defined as [a, b] + [c, d] = [a + c, b + d], Example: If A= [2, 5], B= [1, 3], then A+B= [2+1, 5+3] = [3, 8]

(ii). Subtraction of two intervals is defined as [a, b] − [c, d] = [a − d, b − c], Example. If A= [4, 7], B= [2, 6], then A-B= [4-6, 7-2] = [-2, 5]

(iii). Multiplication of two intervals is defined as [a, b] × [c, d] = [min (ac, ad, bc, bd), max (ac, ad, bc, bd)], Example. If A= [2, 4], B= [3, 5], all the combinations are 2×3=6, 2×5=10, 4×3=12, 4×5=20, Then A×B= [min (6,10,12,20), max (6, 10,12,20)] = [6, 20]

(iv). Division of two intervals is defined as X ÷ Y = X × [1/d, 1/c], provided 0 ∉ [c, d]. Note. Division is undefined if 0 is inside the denominator interval. Example. If A= [6, 10], B= [2, 5], First take reciprocal of B: 1/B= [1/5, 1/2]. Now A÷B= AX 1/B = [6, 10] × [1/5, 1/2], all the combinations: 6×1/5=6/5, 6×1/2=3, 10×1/5=2, 10×1/2=5, then A÷B= [6/5, 5]

1.4. Interval Extension of a Function

- Replace every real variable x with an interval X
- Replace every operation with its interval equivalent
- The result F(X) is guaranteed to contain f(x) for every x ∈ X
- Example: $f(x) = x^2 - 2$, F ([1, 2]) = [1, 4] − 2 = [−1, 2]
- This means: for any x ∈ [1, 2], the value f(x) lies in [−1, 2]

1.5. Interval Derivative

- The interval derivative F′(X) is the interval extension of f′(x) over the interval X
- It gives an enclosure of all possible slope values in the interval
- Example: $f(x) = x^2 - 2$, f′(x) = 2x, so F′ ([1, 2]) = [2, 4]
- Mean Value Theorem for intervals: f(x) ∈ f(m) + F′(X) × (x − m) for all x ∈ X

2. THE INTERVAL NEWTON METHOD

Classical Newton's method is a powerful tool for finding roots, but it has important limitations. It produces only an approximate numerical point rather than a guaranteed bound containing the true solution, and its convergence depends heavily on the choice of the initial guess—if that guess is too far from the actual root, the method may even diverge. Moreover, it cannot rigorously prove whether a root exists (or does not exist) within a given interval. The Interval Newton Method overcomes these shortcomings by working with intervals instead of single values, allowing it to provide verified bounds, ensure reliable convergence behaviour, and rigorously confirm the existence or absence of solutions within a region. Thus, the Interval Newton Method provides reliable and guaranteed root enclosure.

2.1. Derivation from the Mean Value Theorem

By using Lagranges mean value theorem we have $f(x) = f(m) + f'(\xi)(x - m)$ for some ξ ∈ X. If x∗ is a root then f(x∗) = 0, from the above relation , $0 = f(m) + f'(\xi)(x* - m)$ by rearranging we get $x* = \frac{m-f(m)}{f'(\xi)}$. By Replacing f′(ξ) with the interval derivative F′(X) we have the Interval Newton Operator $N(X) = \frac{m-f(m)}{f'(X)}$, Where m(X) = midpoint of the current interval X = $\frac{a+b}{2}$ f(m(X)) = function evaluated at the midpoint, F′(X) = interval extension of f′(x) over X and N(X) is the interval that must contain any root of f in X.

2.2. The Intersection Step

- After computing N(X), take the intersection: $X_{\cdot+1} = X_{\cdot} \cap N(X_{\cdot})$
- The intersection keeps only the part of X confirmed to contain the root.

Case	Result	Meaning
X ∩ N(X) is non-empty	Continue iterating	Root may still exist in X
X ∩ N(X) = ∅ (empty)	STOP — no root here	Proven: no root exists in X
N(X) ⊆ X	STOP — unique root found	Proven: exactly one root in X

2.3. Algorithm of the Interval Newton Method

Input: $f(x)$, initial interval X_0=[a,b], tolerance epsilon
Oupput: interval guaranteed to contain the root
Step 1: Set n = 0
Step 2: Compute $m = \frac{a+b}{2}$ (midpoint of (x_n))
Step 3: Evaluate f(m)
Step 4: Compute F' (X_n) (interval extension of derivative)
Step 5: Compute $N(X_n)$ = m- f(m) /F'(X)
Step 6: Compute $X_{n+1} = X_n \cap N(X_n)$
Step 7: if $x_{n+1} = \Phi$ then no root exists
Step 8: If width (X_{n+1}) <ε then root found, STOP
Step 9: Set n = n + 1 go to Step 2.

2.4. Stopping Criteria

In the interval newton method, we repeat iterations until we are confident that the root is found with sufficient accuracy. This process is stopped based on the following conditions

- Width criterion: $w(X_n) = b_n - a_n < \varepsilon$ (e.g. $\varepsilon = 0.0001$)
- Function value criterion: $|f(m)| < \delta$
- Maximum iterations criterion: $n > N$-max2.5. When $0 \in F'(X)$ — Subdivision Strategy
- "Division is not defined when $0 \in F'(X)$"
- Solution: split X into two sub-intervals and apply the method to each Remark. If the interval contains more than one root, apply subdivision until each sub-interval contains at most one root

3. NUMERICAL EXAMPLES ON INTERVAL NEWTON METHOD

The Interval Newton Method is applied to five different algebraic equations, each analyzed in a consistent and systematic manner. For every example, the process begins by clearly stating the equation along with the chosen initial interval. This is followed by performing the computations step by step for each iteration, ensuring that all intermediate interval evaluations are shown in detail. A complete iteration table is then presented to summarize the progression of the method, and finally, the verified root guaranteed to lie within the computed interval is reported.

3.1 Example

$f(x) = x^2 - 2 = 0$ on $X_0 = [1, 2]$, Simple Quadratic Equation

$f(x)=x^2-2$, Choose initial interval $X_0 = [1, 2]$, $f'(x) = 2x$

Iteration 1: Midpoint: $m_0=1.5$, $f(1.5)=0.25$, $F'(X_0) = [2, 4]$,Interval newton step: $N(x) = m-f(m) /F'(X)$, $N(X_0) = 1.5-[(0.25)/(2,4)] =1.5-[0.0625,0.125]$, $N(X_0)=[1.375 ,1.4375]$, Now $X_1= X_0 \cap N(X) = [1,2]\cap[1.375,1.4375] =[1.375,1.4375]$

Iteration 2: Now repeat with X_1 , $m_1= 1.40625$, $f(m_1)=(-0.0224)$, $F'(X_1) = [2.75,2.875]$, $N(X_1)=[1.4143,1.4139]$, Now $X_2= X1\cap N(X1) =1.375,1.4375] \cap [1.4143, 1.4175]=[1.4143,1.4175]$

Iteration 3: Now repeat X_2, $M_2= 1.4159$, $f(M_2)=0.0047$, $F'(X_2)=[2.8286,2.835]$, $N(X_2)=[1.4143,1.4143]$, Now $x3 = X_2\cap N(X_2) = [1.4143,1.4175]\cap[1.4143,1.4143]$, $X = [1.4143, 1.41\ 43]$

Iteration table (Interval Newton method)

	X n	m	f(m)	F' (Xn)	N(Xn)	Xn+1
0	[1,2]	1.5	0.25	[2,4]	[1.375,1.4375]	[1.375,1.4375]
1	[1.375,1.4375]	1.40625	-0.02246	[2.75,2.875]	[1.4140,1.4143]	[1.4140,1.4143]
2	[1.4140,1.4143]	1.41415	0.000006	[2.8280,2.8286]	[1.41421,1.4142]	[1.4142,1.4142]

The Interval Newton Method converges to the root $x\approx1.4143x$. The width of the interval satisfies $b-a<0.001b$within three iterations, showing that the interval shrinks very rapidly. Hence, the method exhibits fast quadratic convergence and provides highly accurate verified bounds for the root.

3.2 Example

$f(x) = x^3 - x - 2 = 0$ on $X_0 = [1, 2]$ Cubic Polynomial

$f(x)= x^3 - x - 2$, Choose initial interval $X_0 = [1, 2]$, $f'(x) = 3x^2-1$

Midpoint: $m_0=1.5$, $f(1.5)=-0.125$,$F'(X_0) = [2, 11]$,

Newton interval step: $N(X_n) = m-f(m) /F'(X)$

$N(X_0)=[1.5625,1.511]$, Now $X_1= X_0 \cap N(X_0) = [1,2]\cap[1.5625,1.511]] =[1.5625,1.511]$, Now repeat with X_1

Iteration 1: $M_1= 1.5369$, $f(m_1)=0.0933$, $F'(X_1) = [6.3242,5.8520]$,

$N(X_1)=[1.5213,1.5215]$, Now $X_2= X_1\cap N(X_1) =1.5625,1.5113] \cap [1.5213,1.5215] =[1.5625,1.5113]$

Iteration 2: Now repeat X_2, $M_2= 1.5214$, $f(M_2)=0.00002$, $F'(X_2) =[5.94,5.95]$

$N(X_2)=[1.52138,1.52138]$, Now $x_3 = X_2 \cap N(X_2) =[1.5625,1.511] \cap [1.52138,1.52138]] =[1.5625,1.511]$

Iteration table (Newton Raphson method)

n	Xn	m	f(m)	F'(Xn)	N(Xn)	Xn+1
0	[1,2]	1.5	-0.125	[2,11]	[1.51136,1.5625]	[1.51136,1.5625]
1	[1.511,1.5625]	1.53675	0.0933	[5.85,6.32]	[1.5212,1.5216]	[1.5212,1.5216]
2	[1.5213,1.5215]	1.5214	0.00002	[5.94,5.95]	[1.5213,1.5213]	[1.5213,1.5213]

The third iteration produced the interval $X3\approx[1.52138, 1.52138]$, which indicates that the root of the equation is approximately $x\approx1.52138$. The interval reduced smoothly and rapidly during the iterative process, demonstrating the efficiency of the method. An accurate approximation of the root was obtained within only three iterations. The method exhibited fast quadratic convergence, successfully locating the root near $x\approx1.5214$

3.3 Example

$f(x) = \cos(x) - x = 0$ on $X_0 = [0, 1]$ Transcendental Equation

Choose initial interval $X_0 = [0,1]$, $f'(x) = -\sin(x) - 1$

Iteration 1: Midpoint: $m_0=0.5$, $f(0.5)= \cos (0.5)-(0.5)=0.37758$

Derivative interval: $F'(X_0) = [-2,-1]$, Newton interval step: $N(X_n) = m-f(m) /F'(X)$, $N(X)=[0.68879,0.87758]$, Now $X_1= x_0\cap N(X_0) = [0,1]$intersection$[0.68879,0.87758] =[0.68879,0.87758]$, Now repeat with X_1

E-ISSN: 2349-7017
P-ISSN: 2349-7009
https://www.ijiris.com/archives
https://doi.org/10.26562/ijiris.2026.v1205.01

Iteration 2: M_1= 0.78318, $f(m_1)$=-0.07499, F' (X_1) = [-1.77,-1.64], $N(X_1)$=[0.73875,0.74029], Now $X_2 = X_1 \cap N(X_1)$ =[0.68879,0.87758] ∩ [0.73875,0.74029] =[0.73875,0.74029], Now repeat X_2

Iteration 3: M_2= 0.73952, $f(M_2)$=-0.00074, F' (X_2) =[-1.675,-1.673]], $N(X_2)$=[0.73908,0.73909], Now $x_3 = X_2 \cap N(X_2)$ =[0.73875,0.74029] ∩ [0.73908,0.73909] =[0.73908,0.73909]

The interval obtained through successive iterations converges to a very small range containing the root, indicating the effectiveness and accuracy of the method. Hence, the root of the equation is approximately 0.739080. The Interval Newton Method performs efficiently even for non-algebraic equations and provides a guaranteed enclosure for the root, thereby ensuring reliability and numerical stability in the computed solution.

4.4 Example

$f(x) = x^4 - 3x^2 + 2 = 0$ (multiple roots) Higher Degree Polynomial

$f(x) = x^4 - 3x^2 + 2$, Initial interval: X=[-2,2], Evaluate the function : f(-2)=6 (>0), f(-1)=0, f(0)=2 (>0), f(1)=0, f(2)=6 (>0) The function has multiple roots, hence we subdivide the interval, Subdivision of interval: devide the interval into smaller parts: [-2,-1.2] ,[-1.2,-0.8] ,[0.8,1.2], [1.2,2]

Check sign change in each sub interval:

1. Interval [-2,-1.2], f(-2)>0 ,f(-1.2)<0, •Sign change then root exist, •Root~-1.4142
2. Interval [-1.2,-0.8], f(-1.2)<0. , f(-0.8)>0, Sign change then root exist, Root=-1
3. Interval [0.8,1.2], f(0.8)>0 ,f(1.2)<0, Root: 1
4. Interval [1.2,2],f(1.2)<0 ,f(2)>0, Root~1.4141, Thus, the root of equations are. x=-1.4142, -1, 1, 1.4142, By subdividing the initial interval and checking sign changes, all roots of the equations are successfully isolated

4.5 Example

$f(x) = e^x - 3x = 0$ Exponential Equation

$f(x) = e^x - 3x$

Step1: choose the initial interval: Check values: f(0) =1>0, f(1) = e-3=-0.2817<0, Since f(0) f(1) <0 then the root lies in X_0 = [0, 1]

Step 2: Derivative: $f(x) = e^x - 3$

Step 3: Apply interval newton method: Use $N(X_n)$ = m-f(m) /F'(X), $X_{n+1} = X_n \cap N(X_n)$

Iteration1:m_0=0.5, f(0.5)=$e^{0.5}$-1.5=1.6487-1.5 = 0.1487, F' (X_0) = [- 2, - 0.2817], $N(X_0)$ = [0.5743, 1.028], X_1=[0.5743, 1]

Iteration 2: M_1=0.7871, f(0.7871) =-0.164, F'(X_1) =[-1.224, -0. 2817], $N(X_1)$ =[0.5743, 0.653], X_2=[0.5743, 0.653]

Iteration 3: M_2=0.6136, $f(m_2)$=0.006, F'(X_2) =[-1.225, -1. 078], $N(x_2)$ =[0.6185, 0.6192], X_3=[0.6185, 0.6192]

n	Xn	m	f(m)	F'(Xn)	N(Xn)	Xn+1
0	[0, 1]	0.5	0.1487	[-2, -0. 2817]	[0.57, 1]	[0.57, 1]
1	[0.57,1]	0.787	-0.164	[-1.224, -0. 2817]	[0.57, 0.653]	[0.57, 0.653]
2	[0.57, 0.653]	0.613	0.006	[-1.225, -1. 078]	[0.619, 0.619]	[0.619, 0.619]

The convergence of the method is slightly slower due to the exponential nature of the equation. However, the iterative process remains stable and achieves good accuracy within three iterations. Although exponential equations generally lead to slower convergence when compared to polynomial equations, the Interval Newton Method still guarantees the correctness of the computed solution by providing a reliable enclosure of the root throughout the iteration process. All the examples considered in this study demonstrate the rapid convergence behavior of the Interval Newton Method, with the computed intervals shrinking significantly at each iteration. The final interval widths obtained are extremely small, indicating a high degree of accuracy and reliability in the approximated solutions. In Example 5.4, the method efficiently handles the presence of multiple roots through interval subdivision, enabling each root to be isolated and verified separately. In certain cases, exact roots such as ±1\pm 1±1 are obtained, which further highlights the precision, robustness, and effectiveness of the Interval Newton Method in solving nonlinear equations.

4. CONCLUSION

The Interval Newton Method proved to be an effective and reliable technique for solving algebraic, transcendental, and exponential equations. In all examples considered, the method successfully produced guaranteed enclosing intervals containing the true roots, while the interval widths decreased rapidly with successive iterations, demonstrating fast and stable convergence. The method also effectively handled higher-degree equations and multiple roots through interval subdivision, thereby ensuring accurate root isolation and verification. Compared with classical numerical methods, the Interval Newton Method provides greater reliability because of its verification property and controlled error bounds obtained through interval arithmetic. The method offers several important advantages, including guaranteed root containment, effective handling of uncertainty, reduced error propagation, and the ability to identify all roots within a specified interval. It is also highly suitable for verified numerical analysis and scientific computing applications where accuracy and reliability are essential. However, the method has certain limitations, such as overestimation caused by dependency problems, increased computational cost, and the need for interval subdivision when the derivative interval contains zero. Furthermore, interval extensions of some functions may produce wider bounds, reducing computational efficiency for simple problems where classical methods may converge faster. Despite these limitations, the Interval Newton Method has significant real-world applications in engineering, physics, computer science, finance, biology, robotics, optimization, and aerospace science, where mathematically verified and highly reliable solutions are required.

E-ISSN: 2349-7017
P-ISSN: 2349-7009
https://www.ijiris.com/archives
https://doi.org/10.26562/ijiris.2026.v1205.01

Overall, the method represents a powerful numerical approach that overcomes many drawbacks of traditional root-finding techniques by ensuring guaranteed accuracy, stability, and correctness of the computed solutions.

Author contribution Statement

Conceptualization: G.Upender Reddy
Literature Review and Methodology design: M.Varsha, Ch.Shivaranjani
Software: P.Chamanthi
Validation: G.Upender Reddy
Formal Analysis: G.Upender Reddy
Investigation: Ch.Shivaranjani
Resources: G.Upender Reddy
Data Curation: M.Varsha
Writing original draft preparation: G.Upender Reddy
Writing review and Editing: G.Upender Reddy
Visualization: P.Chamanthi
Supervision: G.Upender Reddy
Project Administration: All authors have read and agreed to the published version of the manuscript
Conflict of interest: The authors declare no conflicts of interest.
Data availability statement: Data supporting these findings are available within the article, at https://doi.org/10.26562/ijiris.2026.v1205.01, or upon request.
Publisher's note: AM Publications, India. IJIRIS (International Journal of Innovative Research in Information Security) Journals stays neutral with regard to jurisdictional claims in published maps and institutional affiliations. All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.
https://ampublications-india.com/

REFERENCES

1. G.Alefeld and J. Herzberger, Introduction to Interval Computations. New York, NY, USA: Academic Press, 1983.
2. L.Jaulin, M. Kieffer, O. Didrit, and E. Walter, Applied Interval Analysis. London, U.K.: Springer, 2001. https://doi.org/10.1007/978-1-4471-0249-6
3. R.E.Moore, R. B. Kearfott, and M. J. Cloud, Introduction to Interval Analysis. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics (SIAM), 2009. https://doi.org/10.1137/1.9780898717716
4. A.Neumaier, Interval Methods for Systems of Equations. Cambridge, U.K.: Cambridge University Press, 1990. https://doi.org/10.1017/CBO9780511526473
5. S.S Sastry (2006). An Introduction to Numerical Analysis (2nd ed.). Wiley.